

# VnK Academy Thane

## FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18<sup>th</sup> March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

### PHYSICS

### TEST PAPER WITH ANSWER & SOLUTION

#### SECTION-A

1. Which of the following statements are correct?
- (A) Electric monopoles do not exist whereas magnetic monopoles exist.
- (B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.
- (C) Magnetic field lines are completely confined within a toroid.
- (D) Magnetic field lines inside a bar magnet are not parallel.
- (E)  $\chi = -1$  is the condition for a perfect diamagnetic material, where  $\chi$  is its magnetic susceptibility.

Choose the correct answer from the options given below :

- (1) (C) and (E) only  
 (2) (B) and (D) only  
 (3) (A) and (B) only  
 (4) (B) and (C) only

**Official Ans. by NTA (1)**

- Sol.** Statement (C) is correct because, the magnetic field outside the toroid is zero and they form closed loops inside the toroid itself.

Statement (E) is correct because we know that super conductors are materials inside which the net magnetic field is always zero and they are perfect diamagnetic.

$$\mu_r = 1 + \chi$$

$$\chi = -1$$

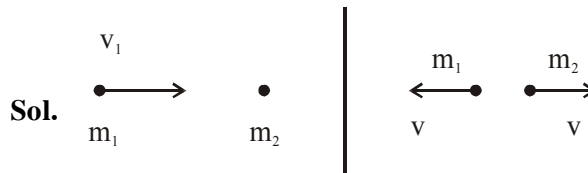
$$\mu_r = 0$$

For superconductors.

2. An object of mass  $m_1$  collides with another object of mass  $m_2$ , which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses  $m_2 : m_1$  is :

- (1) 3 : 1                                      (2) 2 : 1  
 (3) 1 : 2                                      (4) 1 : 1

**Official Ans. by NTA (1)**



$$m_1 v_1 = -m_1 v + m_2 v$$

$$v_1 = -v + \frac{m_2}{m_1} v$$

$$\frac{(v_1 + v)}{v} = \frac{m_2}{m_1}$$

$$e = \frac{2v}{v_1} = 1$$

$$v = \frac{v_1}{2}$$

$$\frac{v_1 + v_1/2}{v_1/2} = \frac{m_2}{m_1}$$

$$3 = \frac{m_2}{m_1}$$

3. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where  $\gamma$  is the ratio of specific heats):

- (1)  $-\gamma \frac{dV}{V}$                                       (2)  $-\gamma \frac{V}{dV}$   
 (3)  $-\frac{1}{\gamma} \frac{dV}{V}$                                       (4)  $\frac{dV}{V}$

**Official Ans. by NTA (1)**

- Sol.**  $PV^\gamma = \text{constant}$   
 Differentiating

$$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

$$\frac{dP}{P} = -\frac{\gamma dV}{V}$$

4. A proton and an  $\alpha$ -particle, having kinetic energies  $K_p$  and  $K_\alpha$ , respectively, enter into a magnetic field at right angles.

The ratio of the radii of trajectory of proton to that of  $\alpha$ -particle is 2 : 1. The ratio of  $K_p : K_\alpha$  is :

- (1) 1 : 8 (2) 8 : 1  
(3) 1 : 4 (4) 4 : 1

**Official Ans. by NTA (4)**

**Sol.**  $r = \frac{mv}{qB} = \frac{p}{qB}$   $\frac{m_\alpha}{m_p} = 4$

$$\frac{r_p}{r_\alpha} = \frac{p_p}{q_p} \frac{q_\alpha}{p_\alpha} = \frac{2}{1}$$

$$\frac{p_p}{p_\alpha} = \frac{2q_p}{q_\alpha} = 2 \left( \frac{1}{2} \right)$$

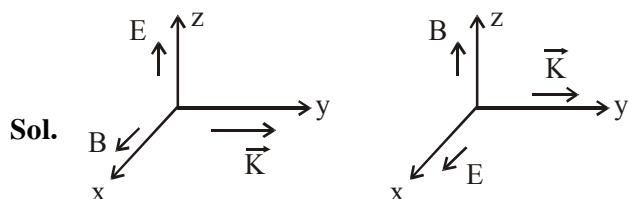
$$\frac{p_p}{p_\alpha} = 1$$

$$\frac{K_p}{K_\alpha} = \frac{p_p^2}{p_\alpha^2} \frac{m_\alpha}{m_p} = (1) (4)$$

5. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field ( $\vec{E}$ ) and magnetic field ( $\vec{B}$ ) components.

- (1)  $E_y, B_y$  or  $E_z, B_z$   
(2)  $E_y, B_x$  or  $E_x, B_y$   
(3)  $E_x, B_z$  or  $E_z, B_x$   
(4)  $E_x, B_y$  or  $E_y, B_x$

**Official Ans. by NTA (3)**



6. Consider a uniform wire of mass  $M$  and length  $L$ . It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is :

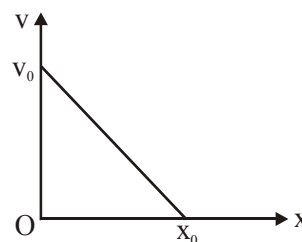
- (1)  $\frac{1}{4} \frac{ML^2}{\pi^2}$  (2)  $\frac{2}{5} \frac{ML^2}{\pi^2}$   
(3)  $\frac{ML^2}{\pi^2}$  (4)  $\frac{1}{2} \frac{ML^2}{\pi^2}$

**Official Ans. by NTA (3)**

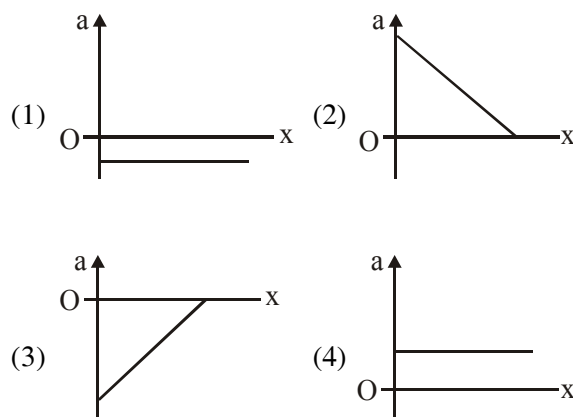
**Sol.**  $\pi r = L \Rightarrow r = \frac{L}{\pi}$

$$I = Mr^2 = \frac{ML^2}{\pi^2}$$

7. The velocity-displacement graph of a particle is shown in the figure.



The acceleration-displacement graph of the same particle is represented by :



**Official Ans. by NTA (3)**

**Sol.**  $v = -\left(\frac{v_0}{x_0}\right)x + v_0$

$$a = \frac{v dv}{dx}$$

$$a = \left[ -\left( \frac{v_0}{x_0} \right) x + v_0 \right] \left[ -\frac{v_0}{x_0} \right]$$

$$a = \left( \frac{v_0}{x_0} \right)^2 x - \frac{v_0^2}{x_0}$$

8. The correct relation between  $\alpha$  (ratio of collector current to emitter current) and  $\beta$  (ratio of collector current to base current) of a transistor is :

$$(1) \beta = \frac{\alpha}{1 + \alpha}$$

$$(2) \alpha = \frac{\beta}{1 - \alpha}$$

$$(3) \beta = \frac{1}{1 - \alpha}$$

$$(4) \alpha = \frac{\beta}{1 + \beta}$$

**Official Ans. by NTA (4)**

**Sol.**  $\alpha = \frac{I_C}{I_E}, \beta = \frac{I_C}{I_B}$

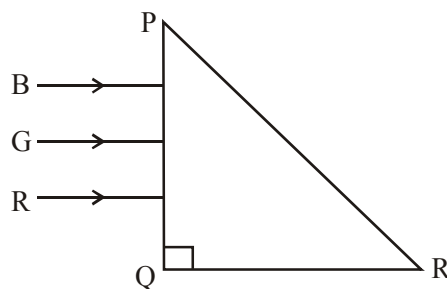
$$I_E = I_B + I_C$$

$$\alpha = \frac{I_C}{I_B + I_C} = \frac{1}{\frac{I_B}{I_C} + 1}$$

$$\alpha = \frac{1}{\frac{1}{\beta} + 1}$$

$$\alpha = \frac{\beta}{1 + \beta}$$

9. Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in figure.

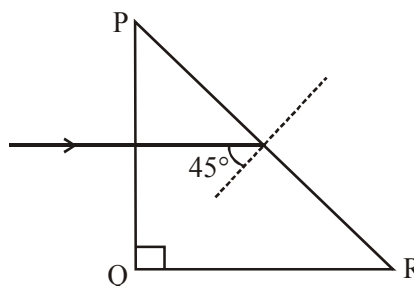


The refractive indices of the material of the prism for red, green and blue wavelengths are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is :

- (1) green (2) red  
(3) blue and green (4) blue

**Official Ans. by NTA (2)**

**Sol.**



Assuming that the right angled prism is an isosceles prism, so the other angles will be  $45^\circ$  each.

$\Rightarrow$  Each incident ray will make an angle of  $45^\circ$  with the normal at face PR.

$\Rightarrow$  The wavelength corresponding to which the incidence angle is less than the critical angle, will pass through PR.

$\Rightarrow \theta_c = \text{critical angle}$

$$\Rightarrow \theta_c = \sin^{-1} \left( \frac{1}{\mu} \right)$$

$\Rightarrow$  If  $\theta_c \geq 45^\circ$   
the light ray will pass

$$\Rightarrow (\theta_c)_{\text{Red}} = \sin^{-1} \left( \frac{1}{1.27} \right) = 51.94^\circ$$

Red will pass.

$$\Rightarrow (\theta_c)_{\text{Green}} = \sin^{-1}\left(\frac{1}{1.42}\right) = 44.76^\circ$$

Green will not pass

$$\Rightarrow (\theta_c)_{\text{Blue}} = \sin^{-1}\left(\frac{1}{1.49}\right) = 42.15^\circ$$

Blue will not pass

$\Rightarrow$  So only red will pass through PR.

- 10.** If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

(Take :  $g = 10 \text{ ms}^{-2}$ , the radius of earth,  $R = 6400 \times 10^3 \text{ m}$ , Take  $\pi = 3.14$ )

- (1) 60 minutes  
(2) does not change  
(3) 1200 minutes  
(4) 84 minutes

**Official Ans. by NTA (4)**

**Sol.** For objects to float

$$mg = m\omega^2 R$$

$\omega$  = angular velocity of earth.

$R$  = Radius of earth

$$\omega = \sqrt{\frac{g}{R}} \quad \dots (1)$$

Duration of day =  $T$

$$T = \frac{2\pi}{\omega} \quad \dots (2)$$

$$\Rightarrow T = 2\pi \sqrt{\frac{R}{g}}$$

$$= 2\pi \sqrt{\frac{6400 \times 10^3}{10}}$$

$$\Rightarrow \frac{T}{60} = 83.775 \text{ minutes}$$

$$\simeq 84 \text{ minutes}$$

- 11.** The decay of a proton to neutron is :

- (1) not possible as proton mass is less than the neutron mass  
(2) possible only inside the nucleus  
(3) not possible but neutron to proton conversion is possible  
(4) always possible as it is associated only with  $\beta^+$  decay

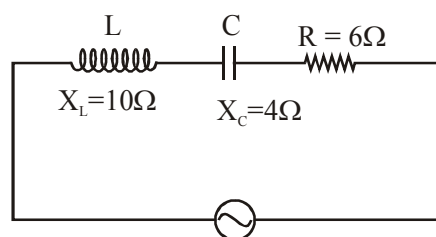
**Official Ans. by NTA (2)**

**Sol.** It is possible only inside the nucleus and not otherwise.

- 12.** In a series LCR circuit, the inductive reactance ( $X_L$ ) is  $10 \Omega$  and the capacitive reactance ( $X_C$ ) is  $4 \Omega$ . The resistance ( $R$ ) in the circuit is  $6 \Omega$ . The power factor of the circuit is :

- (1)  $\frac{1}{2}$  (2)  $\frac{1}{2\sqrt{2}}$   
(3)  $\frac{1}{\sqrt{2}}$  (4)  $\frac{\sqrt{3}}{2}$

**Official Ans. by NTA (3)**



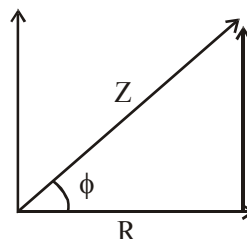
**Sol.**

We know that power factor is  $\cos\phi$ ,

$$\cos\phi = \frac{R}{Z} \quad \dots (1)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \dots (2)$$

$$(\omega L - 1/\omega C)$$



$$\Rightarrow Z = \sqrt{6^2 + (10 - 4)^2}$$

$$\Rightarrow Z = 6\sqrt{2} \mid \cos\phi = \frac{6}{6\sqrt{2}}$$

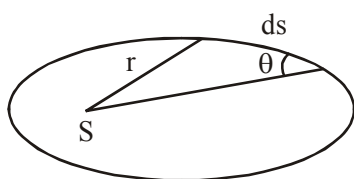
$$\cos\phi = \frac{1}{\sqrt{2}}$$

13. The angular momentum of a planet of mass  $M$  moving around the sun in an elliptical orbit is  $\vec{L}$ . The magnitude of the areal velocity of the planet is :

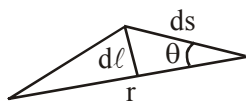
- (1)  $\frac{4L}{M}$  (2)  $\frac{L}{M}$   
(3)  $\frac{2L}{M}$  (4)  $\frac{L}{2M}$

Official Ans. by NTA (4)

Sol.



For small displacement  $ds$  of the planet its area can be written as



$$dA = \frac{1}{2} r d\ell$$

$$= \frac{1}{2} r ds \sin \theta$$

$$\text{A. vel} = \frac{dA}{dt} = \frac{1}{2} r \sin \theta \frac{ds}{dt} = \frac{Vr \sin \theta}{2}$$

$$\frac{dA}{dt} = \frac{1}{2} \frac{mVr \sin \theta}{m} = \frac{L}{2m}$$

14. The function of time representing a simple

harmonic motion with a period of  $\frac{\pi}{\omega}$  is :

- (1)  $\sin(\omega t) + \cos(\omega t)$   
(2)  $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$   
(3)  $\sin^2(\omega t)$   
(4)  $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$

Official Ans. by NTA (4)

Sol. Time period  $T = \frac{2\pi}{\omega'}$

$$\frac{\pi}{\omega} = \frac{2\pi}{\omega'}$$

$\omega' = 2\omega \rightarrow$  Angular frequency of SHM

Option (3)

$$\sin^2 \omega t = \frac{1}{2} (2 \sin^2 \omega t) = \frac{1}{2} (1 - \cos 2\omega t)$$

Angular frequency of  $\left(\frac{1}{2} - \frac{1}{2} \cos 2\omega t\right)$  is  $2\omega$

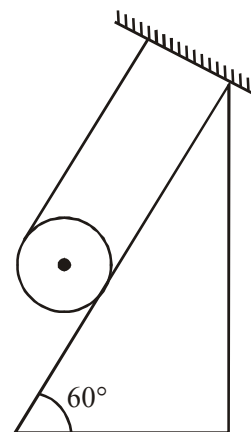
Option (4)

Angular frequency of SHM

$$3 \cos\left(\frac{\pi}{4} - 2\omega t\right) \text{ is } 2\omega.$$

So option (3) & (4) both have angular frequency  $2\omega$  but option (4) is direct answer.

15. A solid cylinder of mass  $m$  is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :

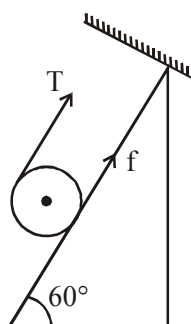


[The coefficient of static friction,  $\mu_s$ , is 0.4]

- (1)  $\frac{7}{2}mg$  (2)  $5mg$   
(3)  $\frac{mg}{5}$  (4) 0

Official Ans. by NTA (3)

Sol.



Let's take solid cylinder is in equilibrium

$$T + f = mg \sin 60 \quad \dots(i)$$

$$TR - fR = 0 \quad \dots(ii)$$

Solving we get

$$T = f_{\text{req}} = \frac{mg \sin \theta}{2}$$

But limiting friction < required friction

$$\mu mg \cos 60^\circ < \frac{mg \sin 60^\circ}{2}$$

$\therefore$  Hence cylinder will not remain in equilibrium

Hence  $f = \text{kinetic}$

$$= \mu_k N$$

$$= \mu_k mg \cos 60^\circ$$

$$= \frac{mg}{5}$$

16. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is :

(1)  $\frac{L}{R} \ln 5$  (2) infinite

(3)  $\frac{L}{R} \ln 2$  (4)  $\frac{L}{R} \ln 10$

Official Ans. by NTA (3)

Sol. Magnetic energy =  $\frac{1}{2} Li^2 = 25\%$

$$ME \Rightarrow 25\% \Rightarrow i = \frac{i_0}{2}$$

$$i = i_0 (1 - e^{-Rt/L}) \text{ for charging}$$

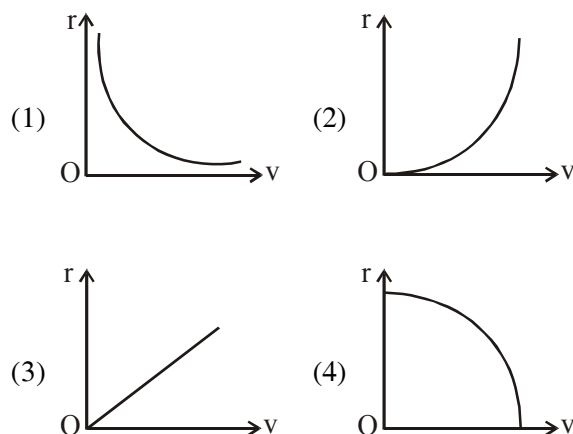
$$t = \frac{L}{R} \ln 2$$

17. A particle of mass m moves in a circular orbit

under the central potential field,  $U(r) = \frac{-C}{r}$ ,

where C is a positive constant.

The correct radius - velocity graph of the particle's motion is :



Official Ans. by NTA (1)

Sol.  $U = -\frac{C}{r}$

$$F = -\frac{dU}{dr} = -\frac{C}{r^2}$$

$$|F| = \frac{mv^2}{r}$$

$$\frac{C}{r^2} = \frac{mv^2}{r}$$

$$v^2 \propto \frac{1}{r}$$

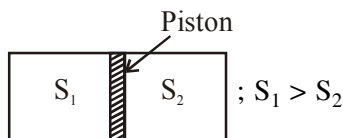
18. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is  $S_1$  and that of the other part is  $S_2$ . Given that  $S_1 > S_2$ . If the piston is removed then the total entropy of the system will be :

(1)  $S_1 \times S_2$  (2)  $S_1 - S_2$

(3)  $\frac{S_1}{S_2}$  (4)  $S_1 + S_2$

Official Ans. by NTA (4)

Sol.



After piston is removed



19. Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :  
(Molecular weight of oxygen is 32 g/mol;  $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$ )

- (1)  $\sqrt{\frac{3}{3}}$  (2)  $\sqrt{\frac{8}{3}}$   
(3)  $\sqrt{\frac{3\pi}{8}}$  (4)  $\sqrt{\frac{8\pi}{3}}$

Official Ans. by NTA (3)

Sol.  $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}}$$

$$\frac{v_{\text{rms}}}{v_{\text{avg}}} = \sqrt{\frac{3\pi}{8}}$$

20. The speed of electrons in a scanning electron microscope is  $1 \times 10^7 \text{ ms}^{-1}$ . If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of:

- (1) 1837 (2)  $\frac{1}{1837}$   
(3)  $\sqrt{1837}$  (4)  $\frac{1}{\sqrt{1837}}$

Official Ans. by NTA (1)

Sol. Resolving power (RP)  $\propto \frac{1}{\lambda}$

$$\lambda = \frac{h}{P} = \frac{h}{mv}$$

$$\text{So (RP)} \propto \frac{mv}{h}$$

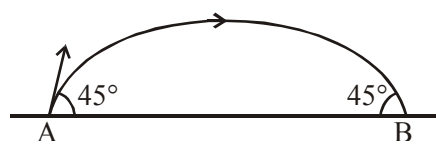
$$\text{RP} \propto P$$

$$\text{RP} \propto mv$$

$$\text{RP} \propto m$$

### SECTION-B

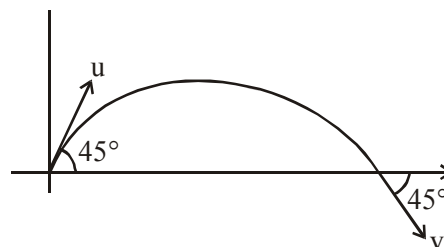
1. The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is  $5\sqrt{2} \text{ ms}^{-1}$  and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is  $x \times 10^{-2} \text{ kgms}^{-1}$ . The value of x, to the nearest integer, is \_\_\_\_\_.

Official Ans. by NTA (5)

Sol.



$$|\vec{u}| = |\vec{v}| \quad \dots (1)$$

$$\vec{u} = u \cos 45^\circ \hat{i} + u \sin 45^\circ \hat{j} \quad \dots (2)$$

$$\vec{v} = v \cos 45^\circ \hat{i} - v \sin 45^\circ \hat{j} \quad \dots (3)$$

$$|\Delta \vec{P}| = |m(\vec{v} - \vec{u})| \quad \dots (4)$$

$$\Delta P = 2mu \sin 45^\circ$$

$$= 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$= 50 \times 10^{-3}$$

$$= 5 \times 10^{-2}$$

2. A ball of mass 4 kg, moving with a velocity of  $10 \text{ ms}^{-1}$ , collides with a spring of length 8 m and force constant  $100 \text{ Nm}^{-1}$ . The length of the compressed spring is  $x$  m. The value of  $x$ , to the nearest integer, is\_\_\_\_\_.

**Official Ans. by NTA (6)**

**Sol.** Let's say the compression in the spring by :  $y$ .  
So, by work energy theorem we have

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}ky^2$$

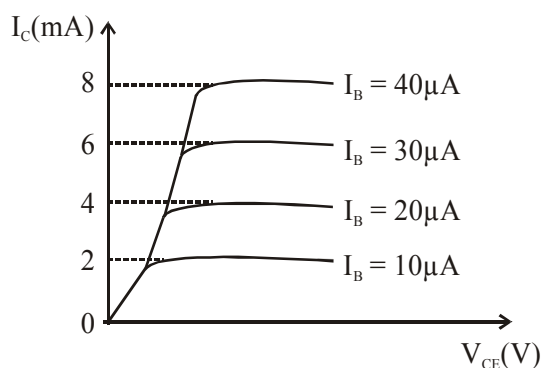
$$\Rightarrow y = \sqrt{\frac{m}{k}} \cdot v$$

$$\Rightarrow y = \sqrt{\frac{4}{100}} \times 10$$

$$\Rightarrow y = 2\text{m}$$

$$\Rightarrow \text{final length of spring} \\ = 8 - 2 = 6\text{m}$$

3. The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is

**Official Ans. by NTA (200)**

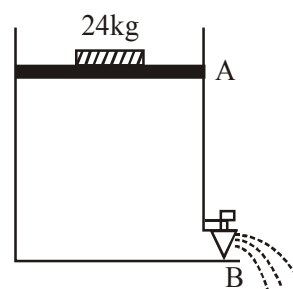
**Sol.**  $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2 \times 10^{-3}}{10 \times 10^{-6}}$

$$\beta = \frac{1}{5} \times 10^3$$

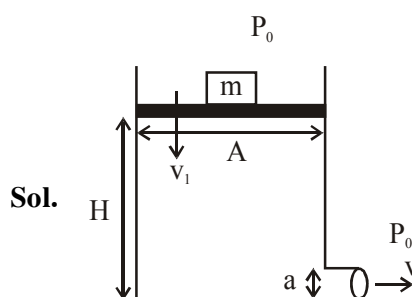
$$\beta = 2 \times 10^2$$

$$\beta = 200$$

4. Consider a water tank as shown in the figure. It's cross-sectional area is  $0.4 \text{ m}^2$ . The tank has an opening B near the bottom whose cross-section area is  $1 \text{ cm}^2$ . A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is  $v \text{ ms}^{-1}$ . The value of  $v$ , to the nearest integer, is\_\_\_\_. [Take value of  $g$  to be  $10 \text{ ms}^{-2}$ ]



**Official Ans. by NTA (3)**



$$m = 24 \text{ kg}$$

$$A = 0.4 \text{ m}^2$$

$$a = 1 \text{ cm}^2$$

$$H = 40\text{cm}$$

Using Bernoulli's equation

$$\Rightarrow \left( P_0 + \frac{mg}{A} \right) + \rho gH + \frac{1}{2}\rho v_1^2 = 0$$

$$= P_0 + 0 + \frac{1}{2}\rho v^2 \quad \dots (1)$$

$$\Rightarrow \text{Neglecting } v_1$$



$$\Rightarrow v = \sqrt{2gH + \frac{2mg}{A\rho}}$$

$$\Rightarrow v = \sqrt{8 + 1.2}$$

$$\Rightarrow v = 3.033 \text{ m/s}$$

$$\Rightarrow v \approx 3 \text{ m/s}$$

5. A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.

(i) ground level

(ii) a height of 5 m.

The increase in antenna range in case (ii) relative to case (i) is n%.

The value of n, to the nearest integer, is .

**Official Ans. by NTA (50)**

**Sol.** Range =  $\sqrt{2Rh}$

Range (i) =  $\sqrt{2Rh}$

Range (ii) =  $\sqrt{2Rh} + \sqrt{2Rh'}$

where h = 20 m & h' = 5m

$$\text{Ans} = \frac{\sqrt{2Rh'}}{\sqrt{2Rh}} \times 100\% = \frac{\sqrt{5}}{\sqrt{20}} \times 100\% = 50\%$$

6. The radius of a sphere is measured to be  $(7.50 \pm 0.85)$  cm. Suppose the percentage error in its volume is x. The value of x, to the nearest x, is\_\_\_\_\_.

**Official Ans. by NTA (34)**

**Sol.**  $\therefore v = \frac{4}{3}\pi r^3$

taking log & then differentiate

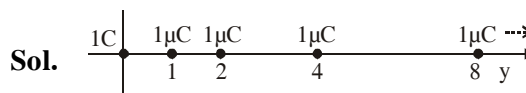
$$\frac{dV}{V} = 3 \frac{dr}{r}$$

$$= \frac{3 \times 0.85}{7.5} \times 100\% = 34\%$$

7. An infinite number of point charges, each carrying  $1 \mu\text{C}$  charge, are placed along the y-axis at  $y = 1 \text{ m}, 2 \text{ m}, 4 \text{ m}, 8 \text{ m}, \dots$ . The total force on a  $1 \text{ C}$  point charge, placed at the origin, is  $x \times 10^3 \text{ N}$ . The value of x, to the nearest integer, is\_\_\_\_\_.

[Take  $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$ ]

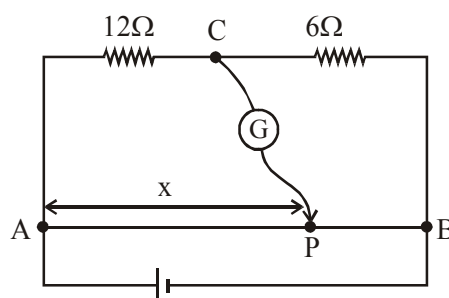
**Official Ans. by NTA (12)**



$$F = k(1\text{C})(1\mu\text{C}) \left[ 1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \dots \right]$$

$$= 9 \times 10^3 \left[ \frac{1}{1 - \frac{1}{4}} \right] = 12 \times 10^3 \text{ N}$$

8. Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x, to the nearest integer, is

**Official Ans. by NTA (48)**

**Sol.** In Balanced conditions

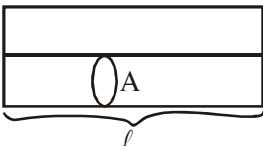
$$\frac{12}{6} = \frac{x}{72 - x}$$

$$x = 48 \text{ cm}$$

9. Two wires of same length and thickness having specific resistances  $6\Omega \text{ cm}$  and  $3\Omega \text{ cm}$  respectively are connected in parallel. The effective resistivity is  $\rho \Omega \text{ cm}$ . The value of  $\rho$ , to the nearest integer, is\_\_\_\_\_.

**Official Ans. by NTA (4)**

**Sol.**  $\therefore$  in parallel

$$R_{\text{net}} = \frac{R_1 R_2}{R_1 + R_2}$$


$$\frac{\rho \ell}{2A} = \frac{\rho_1 \frac{\ell}{A} \times \rho_2 \frac{\ell}{A}}{\rho_1 \frac{\ell}{A} + \rho_2 \frac{\ell}{A}}$$

$$\frac{\rho}{2} = \frac{6 \times 3}{6 + 3} = 2$$

$$\rho = 4$$

10. A galaxy is moving away from the earth at a speed of  $286 \text{ kms}^{-1}$ . The shift in the wavelength of a red line at  $630 \text{ nm}$  is  $x \times 10^{-10} \text{ m}$ . The value of  $x$ , to the nearest integer, is\_\_\_\_\_.

[Take the value of speed of light  $c$ , as  $3 \times 10^8 \text{ ms}^{-1}$ ]

**Official Ans. by NTA (6)**

**Sol.**  $\frac{\Delta \lambda}{\lambda} c = v$

$$\Delta \lambda = \frac{v}{c} \times \lambda = \frac{286}{3 \times 10^5} \times 630 \times 10^{-9} = 6 \times 10^{-10}$$

# VnK Academy Thane

## FINAL JEE-MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18<sup>th</sup> March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

### CHEMISTRY

### TEST PAPER WITH ANSWER & SOLUTION

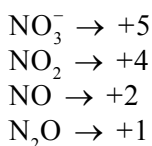
#### SECTION-A

1. The oxidation states of nitrogen in NO, NO<sub>2</sub>, N<sub>2</sub>O and NO<sub>3</sub><sup>-</sup> are in the order of :

- (1) NO<sub>3</sub><sup>-</sup> > NO<sub>2</sub> > NO > N<sub>2</sub>O
- (2) NO<sub>2</sub> > NO<sub>3</sub><sup>-</sup> > NO > N<sub>2</sub>O
- (3) N<sub>2</sub>O > NO<sub>2</sub> > NO > NO<sub>3</sub><sup>-</sup>
- (4) NO > NO<sub>2</sub> > N<sub>2</sub>O > NO<sub>3</sub><sup>-</sup>

**Official Ans. by NTA (1)**

- Sol.** The oxidation states of Nitrogen in following molecules are as follows



2. In basic medium, H<sub>2</sub>O<sub>2</sub> exhibits which of the following reactions ?

- (A) Mn<sup>2+</sup> → Mn<sup>4+</sup>
- (B) I<sub>2</sub> → I<sup>-</sup>
- (C) PbS → PbSO<sub>4</sub>

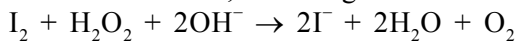
Choose the most appropriate answer from the options given below :

- (1) (A), (C) only
- (2) (A) only
- (3) (B) only
- (4) (A), (B) only

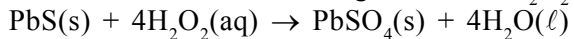
**Official Ans. by NTA (4)**

- Sol.** In basic medium, oxidising action of H<sub>2</sub>O<sub>2</sub>.  
Mn<sup>2+</sup> + H<sub>2</sub>O<sub>2</sub> → Mn<sup>4+</sup> + 2OH<sup>-</sup>

In basic medium, reducing action of H<sub>2</sub>O<sub>2</sub>



In acidic medium, oxidising action of H<sub>2</sub>O<sub>2</sub>.

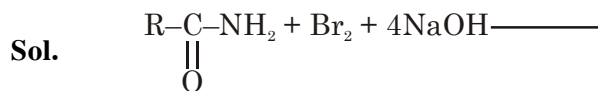


Hence correct option (4)

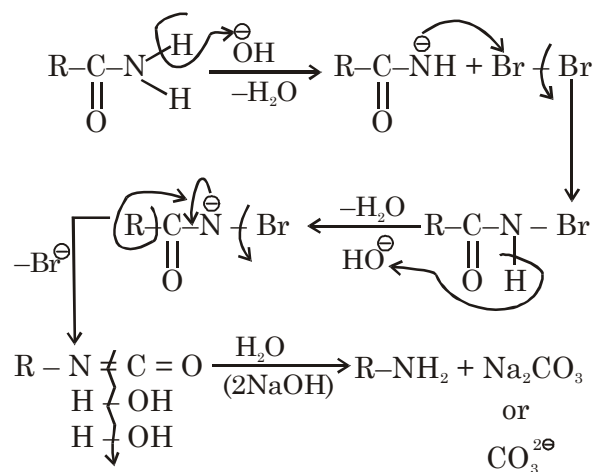
3. In the reaction of hypobromite with amide, the carbonyl carbon is lost as :

- (1) CO<sub>3</sub><sup>2-</sup>
- (2) HCO<sub>3</sub><sup>-</sup>
- (3) CO<sub>2</sub>
- (4) CO

**Official Ans. by NTA (1)**



#### Mechanism



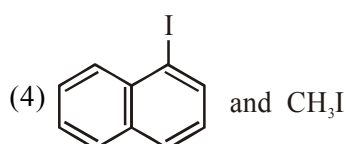
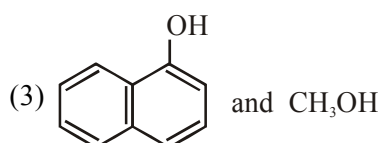
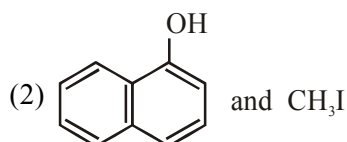
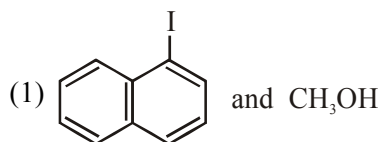
4. The oxide that shows magnetic property is :

- (1) SiO<sub>2</sub>
- (2) Mn<sub>3</sub>O<sub>4</sub>
- (3) Na<sub>2</sub>O
- (4) MgO

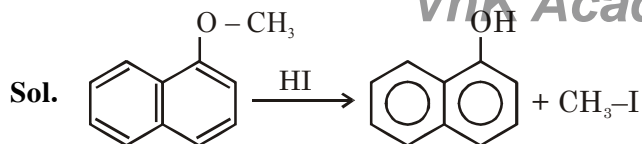
**Official Ans. by NTA (2)**

- Sol.** Mn<sub>3</sub>O<sub>4</sub> shows magnetic properties.

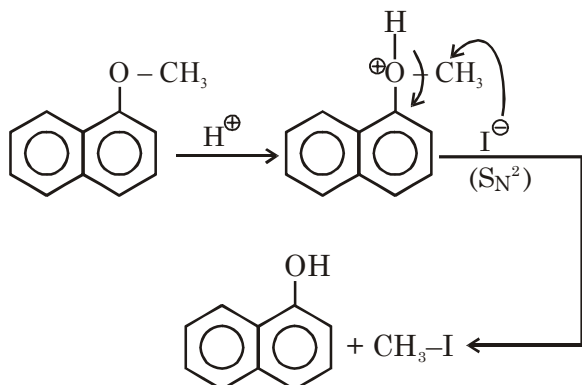
5. Main Products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are:



**Official Ans. by NTA (2)**



**Mechanism**



6. Deficiency of vitamin K causes :
- (1) Increase in blood clotting time
  - (2) Increase in fragility of RBC's
  - (3) Cheilosis
  - (4) Decrease in blood clotting time

**Official Ans. by NTA (1)**

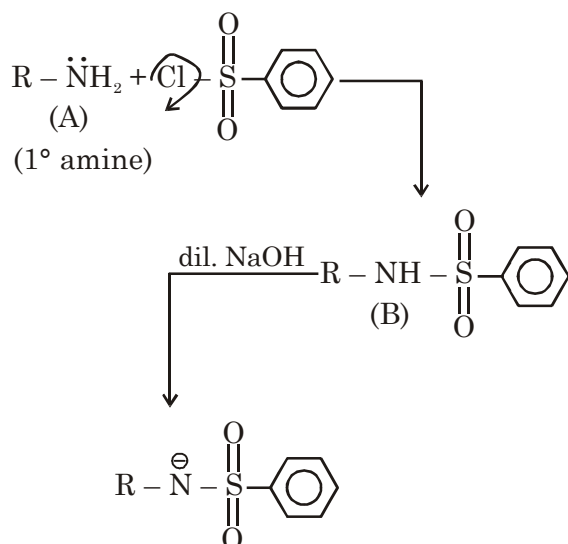
**Sol.** Due to deficiency of Vitamin K causes increases in blood clotting time.

**Note :** Vitamin K related to blood factor.

7. An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is :
- (1)  $\text{C}_6\text{H}_5\text{-N-(CH}_3)_2$
  - (2)  $\text{C}_6\text{H}_5\text{-NHCH}_2\text{CH}_3$
  - (3)  $\text{C}_6\text{H}_5\text{-CH}_2\text{NHCH}_3$
  - (4)  $\text{C}_6\text{H}_5\text{-CH(CH}_3\text{)-NH}_2$

**Official Ans. by NTA (4)**

**Sol.** Hinsberg reagent (Benzene sulphonyl chloride) gives reaction product with  $1^\circ$  amine and it is soluble in dil. NaOH.



8. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. The elements X, Y and Z, respectively, are :

- (1) chlorine, lithium and sodium
- (2) argon, lithium and sodium
- (3) argon, chlorine and sodium
- (4) neon, sodium and chlorine

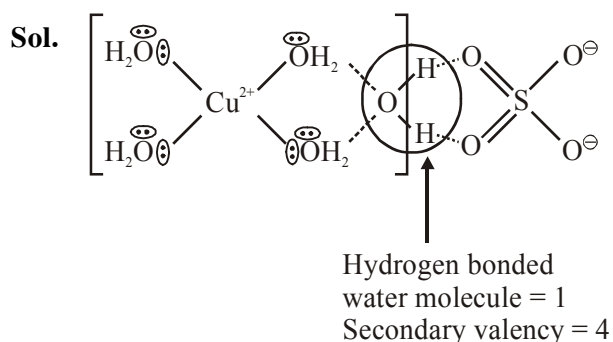
**Official Ans. by NTA (3)**

**Sol.** The 1<sup>st</sup> IE order of 3<sup>rd</sup> period is  $\text{Na} < \text{Al} < \text{Mg} < \text{Si} < \text{P} < \text{Cl} < \text{Ar}$   
X & Y are Ar & Cl  
Z is sodium (Na).

9. The secondary valency and the number of hydrogen bonded water molecule(s) in  $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ , respectively, are :

- (1) 6 and 4
- (2) 4 and 1
- (3) 6 and 5
- (4) 5 and 1

**Official Ans. by NTA (2)**



10. Given below are two statements :

Statement I : Bohr's theory accounts for the stability and line spectrum of  $\text{Li}^+$  ion.

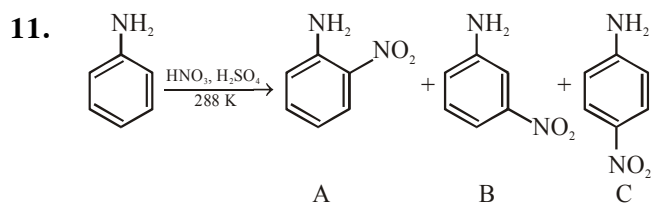
Statement II : Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Both statement I and statement II are true.
- (2) Statement I is false but statement II is true.
- (3) Both statement I and statement II are false.
- (4) Statement I is true but statement II is false.

**Official Ans. by NTA (2)**

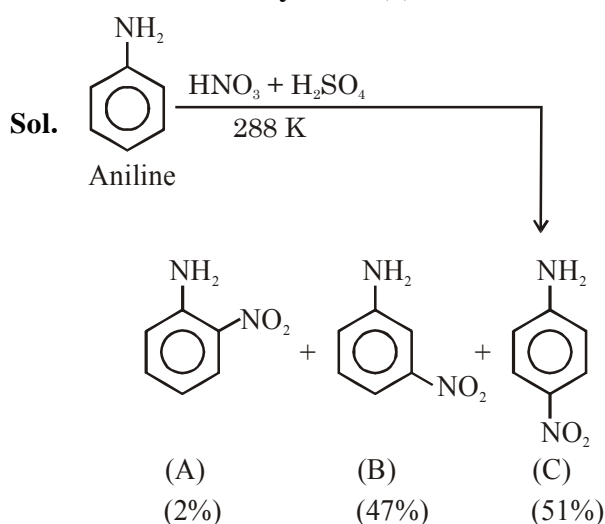
**Sol.** Statement-I is false since Bohr's theory accounts for the stability and spectrum of single electronic species (eg :  $\text{He}^+$ ,  $\text{Li}^{2+}$  etc)  
Statement II is true.



Consider the given reaction, percentage yield of :

- (1)  $C > A > B$  (2)  $B > C > A$   
(3)  $A > C > B$  (4)  $C > B > A$

**Official Ans. by NTA (4)**



% yield order  $\Rightarrow C > B > A$

**12.** The charges on the colloidal  $\text{CdS}$  sol and  $\text{TiO}_2$  sol are, respectively :

- (1) positive and positive  
(2) positive and negative  
(3) negative and negative  
(4) negative and positive

**Official Ans. by NTA (4)**

**Sol.**  $\text{CdS}$  sol  $\rightarrow$  -ve sol  
 $\text{TiO}_2$  sol  $\rightarrow$  +ve sol

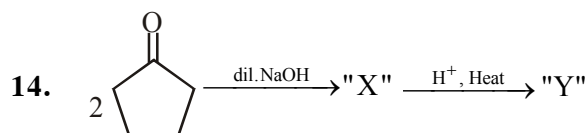
**13.** Match List - I with List - II :

- | List - I                                   | List - II           |
|--|---------------------|
| (Class of Chemicals)                       | (Example)           |
| (a) Antifertility drug                     | (i) Meprobamate     |
| (b) Antibiotic                             | (ii) Alitame        |
| (c) Tranquilizer                           | (iii) Norethindrone |
| (d) Artificial Sweetener                   | (iv) Salvarsan      |
| (1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i) |                     |
| (2) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i) |                     |
| (3) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii) |                     |
| (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii) |                     |

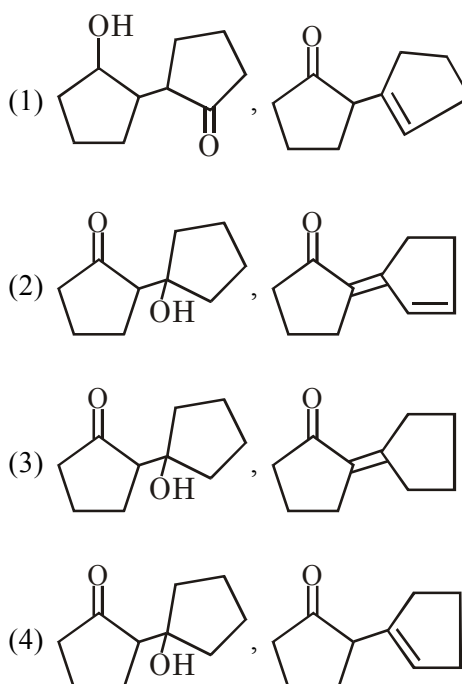
**Official Ans. by NTA (3)**

**Sol.** (A) Antifertility drug  $\rightarrow$  (iii) Nor ethindrone  
(B) Antibiotic  $\rightarrow$  (iv) Salvarsan  
(C) Tranquilizer  $\rightarrow$  (i) Meprobamate  
(D) Artificial sweetener  $\rightarrow$  (ii) Alitame

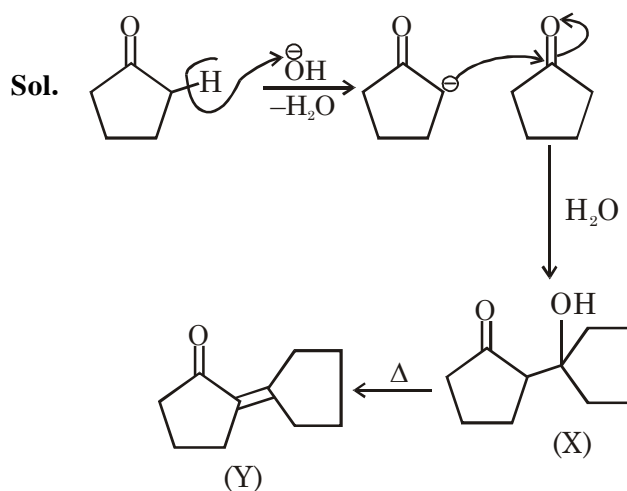
**Ans.** A-iii, B-iv, C-i, D-ii



Consider the above reaction, the product 'X' and 'Y' respectively are :



**Official Ans. by NTA (3)**



15. Match list-I with list-II :

**List-I                      List-II**

- |        |                                    |
|--------|------------------------------------|
| (a) Be | (i) Treatment of cancer            |
| (b) Mg | (ii) Extraction of metals          |
| (c) Ca | (iii) Incendiary bombs and signals |
| (d) Ra | (iv) Windows of X-ray tubes        |
|        | (v) Bearings for motor engines.    |

Choose the most appropriate answer the option given below :

- (1) a-iv, b-iii, c-i, d-ii  
 (2) a-iv, b-iii, c-ii, d-i  
 (3) a-iii, b-iv, c-v, d-ii  
 (4) a-iii, b-iv, c-ii, d-v

**Official Ans. by NTA (2)**

- Sol.** (a) Be → it is used in the Windows of X-ray tubes  
 (b) Mg → it is used in the Incendiary bombs and signals  
 (c) Ca → it is used in the Extraction of metals  
 (d) Ra → it is used in the Treatment of cancer

16. Given below are two statements :

**Statement I :**  $C_2H_5OH$  and  $AgCN$  both can generate nucleophile.

**Statement II :**  $KCN$  and  $AgCN$  both will generate nitrile nucleophile with all reaction conditions.

Choose the most appropriate option :

- (1) Statement I is true but statement II is false  
 (2) Both statement I and statement II are true  
 (3) Statement I is false but statement II is true  
 (4) Both statement I and statement II are false

**Official Ans. by NTA (1)**

17. Given below are two statements :

**Statement I :** Non-biodegradable wastes are generated by the thermal power plants.

**Statement II :** Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the most appropriate answer from the option given below :

- (1) Both statement I and statement II are false  
 (2) Statement I is true but statement II is false  
 (3) Statement I is false but statement II is true  
 (4) Both statement I and statement II are true.

**Official Ans. by NTA (4)**

- Sol.** Non-biodegradable wastes are generated by the thermal power plants which produces fly ash. Detergents which are biodegradable causes problem called eutrophication which kills animal life by depriving it of oxygen.

18. Match list-I with list-II :

**List-I                      List-II**

- |             |                             |
|-------------|-----------------------------|
| (a) Mercury | (i) Vapour phase refining   |
| (b) Copper  | (ii) Distillation refining  |
| (c) Silicon | (iii) Electrolytic refining |
| (d) Nickel  | (iv) Zone refining          |

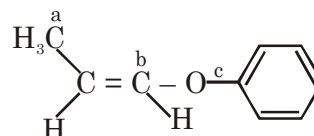
Choose the most appropriate answer from the option given below :

- (1) a-i, b-iv, c-ii, d-iii    (2) a-ii, b-iii, c-i, d-iv  
 (3) a-ii, b-iii, c-iv, d-i    (4) a-ii, b-iv, c-iii, d-i

**Official Ans. by NTA (3)**

- Sol.** (a) Mercury → Distillation refining  
 (b) Copper → Electrolytic refining  
 (c) Silicon → Zone refining  
 (d) Nickel → Vapour phase refining

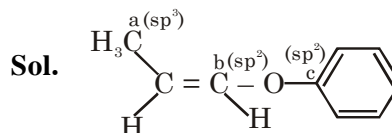
19. In the following molecules,



Hybridisation of carbon a, b and c respectively are :

- (1)  $sp^3$ ,  $sp$ ,  $sp$                       (2)  $sp^3$ ,  $sp^2$ ,  $sp$   
 (3)  $sp^3$ ,  $sp^2$ ,  $sp^2$                       (4)  $sp^3$ ,  $sp$ ,  $sp^2$

**Official Ans. by NTA (3)**



20. A hard substance melts at high temperature and is an insulator in both solid and in molten state.

This solid is most likely to be a / an :

- (1) Ionic solid                      (2) Molecular solid  
 (3) Metallic solid                      (4) Covalent solid

**Official Ans. by NTA (4)**

**Sol.** Covalent or network solid have very high melting point and they are insulators in their solid and molten form.

### SECTION-B

1. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is \_\_\_\_\_ min. (Round off to the Nearest integer)

[Use :  $\ln 2 = 0.69$ ,  $\ln 10 = 2.3$ ]

**Official Ans. by NTA (10)**

**Sol.** 
$$\frac{t_{99.9\%}}{t_{50\%}} = \frac{\frac{1}{K} \ln \frac{100}{0.1}}{\frac{1}{K} \ln 2}$$

$$= \frac{\ln 1000}{\ln 2} \times t_{50\%}$$

$$= \frac{3 \ln 10}{\ln 2} \times 1$$

$$= \frac{3 \times 2.3}{0.69} = 10$$

2. The molar conductivities at infinite dilution of barium chloride, sulphuric acid and hydrochloric acid are 280, 860 and 426  $\text{Scm}^2 \text{mol}^{-1}$  respectively. The molar conductivity at infinite dilution of barium sulphate is \_\_\_\_\_  $\text{Scm}^2 \text{mol}^{-1}$  (Round off to the Nearest Integer).

**Official Ans. by NTA (288)**

**Sol.** From Kohlrausch's law

$$\Lambda_m^\infty(\text{BaSO}_4) = \lambda_m^\infty(\text{Ba}^{2+}) + \lambda_m^\infty(\text{SO}_4^{2-})$$

$$\Lambda_m^\infty(\text{BaSO}_4) = \Lambda_m^\infty(\text{BaCl}_2) + \Lambda_m^\infty(\text{H}_2\text{SO}_4) - 2 \Lambda_m^\infty(\text{HCl})$$

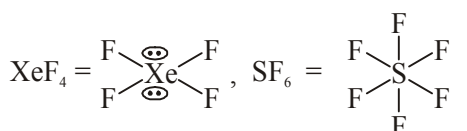
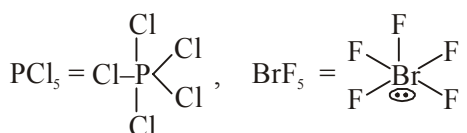
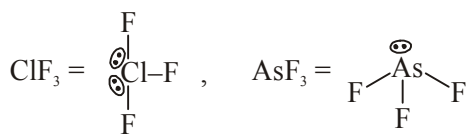
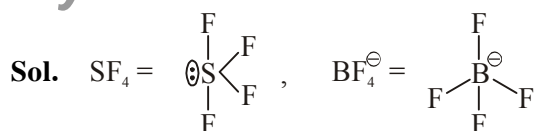
$$= 280 + 860 - 2(426)$$

$$= 288 \text{ Scm}^2 \text{mol}^{-1}$$

3. The number of species below that have two lone pairs of electrons in their central atom is \_\_\_\_\_ (Round off to the Nearest integer)

$\text{SF}_4$ ,  $\text{BF}_4^-$ ,  $\text{ClF}_3$ ,  $\text{AsF}_3$ ,  $\text{PCl}_5$ ,  $\text{BrF}_5$ ,  $\text{XeF}_4$ ,  $\text{SF}_6$

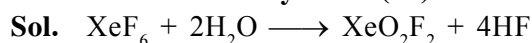
**Official Ans. by NTA (2)**



Two l.p. on central atom is =  $\text{ClF}_3$ ,  $\text{XeF}_4$

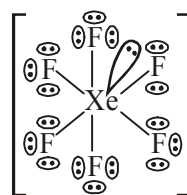
4. A xenon compound 'A' upon partial hydrolysis gives  $\text{XeO}_2\text{F}_2$ . The number of lone pair of electrons present in compound A is \_\_\_\_\_ (Round off to the Nearest integer)

**Official Ans. by NTA (19)**



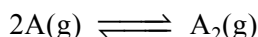
(A) (Limited water)

Structure of 'A'



Total l.p. on (A) = 19

5. The gas phase reaction



at 400 K has  $\Delta G^\circ = + 25.2 \text{ kJ mol}^{-1}$ .

The equilibrium constant  $K_C$  for this reaction is \_\_\_\_\_  $\times 10^{-2}$ . (Round off to the Nearest integer)

[Use :  $R = 8.3 \text{ J mol}^{-1}\text{K}^{-1}$ ,  $\ln 10 = 2.3$ ]

$\log_{10} 2 = 0.30$ ,  $1 \text{ atm} = 1 \text{ bar}$ ]

[antilog  $(-0.3) = 0.501$ ]

**Official Ans. by NTA (166)****Official Ans. by ALLEN (2)****Sol.** Using formula

$$\Delta_r G^0 = -RT \ln K_p$$

$$25200 = -2.3 \times 8.3 \times 400 \log(K_p)$$

$$K_p = 10^{-3.3} = 10^{-3} \times 0.501$$

$$= 5.01 \times 10^{-4} \text{ Bar}^{-1}$$

$$= 5.01 \times 10^{-9} \text{ Pa}^{-1}$$

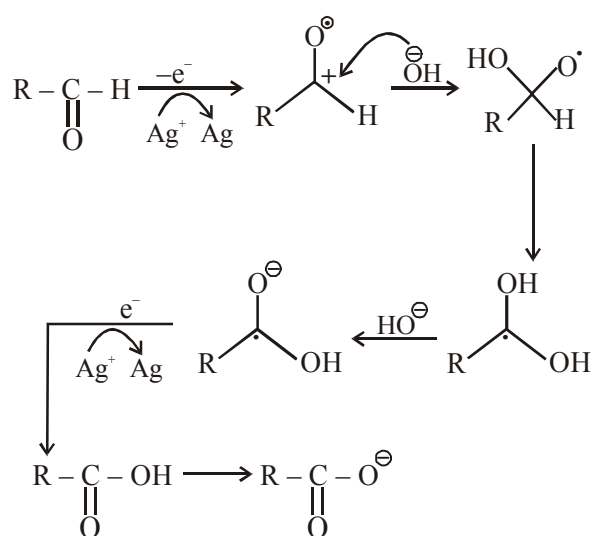
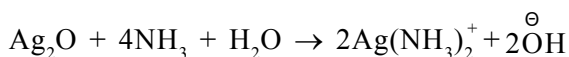
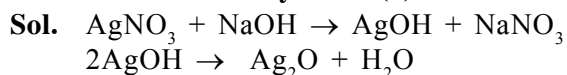
$$= \frac{K_c}{8.3 \times 400}$$

$$K_c = 1.66 \times 10^{-5} \text{ m}^3/\text{mole}$$

$$= 1.66 \times 10^{-2} \text{ L/mol}$$

$$\text{Ans} = 2$$

6. In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula  $[\text{Ag}(\text{NH}_3)_2]^+$  per aldehyde group to form silver mirror is \_\_\_\_\_. (Round off to the Nearest integer)

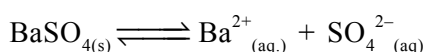
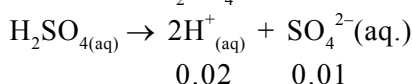
**Official Ans. by NTA (2)**Total  $2e^-$  transfer to Tollen's reagent

7. The solubility of  $\text{CdSO}_4$  in water is  $8.0 \times 10^{-4} \text{ mol L}^{-1}$ . Its solubility in  $0.01 \text{ M H}_2\text{SO}_4$  solution is \_\_\_\_\_  $\times 10^{-6} \text{ mol L}^{-1}$ . (Round off to the Nearest integer) (Assume that solubility is much less than  $0.01 \text{ M}$ )

**Official Ans. by NTA (64)****Sol.** In pure water,

$$K_{sp} = S^2 = (8 \times 10^{-4})^2$$

$$= 64 \times 10^{-8}$$

In  $0.01 \text{ M H}_2\text{SO}_4$ 

$$x \qquad \qquad x \qquad (x + 0.01)$$

$$K_{sp} = x(x + 0.01)$$

$$= 64 \times 10^{-8}$$

$$x + 0.01 \cong 0.01 \text{ M}$$

$$\text{So, } x(0.01) = 64 \times 10^{-8}$$

$$x = 64 \times 10^{-6} \text{ M}$$

8. A solute dimerizes in water. The boiling point of a 2 molar solution of A is  $100.52^\circ\text{C}$ . The percentage association of A is \_\_\_\_\_.

(Round off to the Nearest integer)

[Use :  $K_b$  for water =  $0.52 \text{ K kg mol}^{-1}$ ]Boiling point of water =  $100^\circ\text{C}$ ]**Official Ans. by NTA (50)****Official Ans. by ALLEN (100)****Sol.**  $\Delta T_b = T_b - T_b^0$ 

$$100.52 - 100$$

$$= 0.52^\circ\text{C}$$

$$i = \left(1 - \frac{\alpha}{2}\right)$$

$$\therefore \Delta T_b = i K_b \times m$$

$$0.52 = \left(1 - \frac{\alpha}{2}\right) \times 0.52 \times 2$$

$$\alpha = 1$$

So, percentage association = 100%.



9. 10.0 ml of  $\text{Na}_2\text{CO}_3$  solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings.

4.8 ml, 4.9 ml, 5.0 ml, 5.0 ml and 5.0 ml

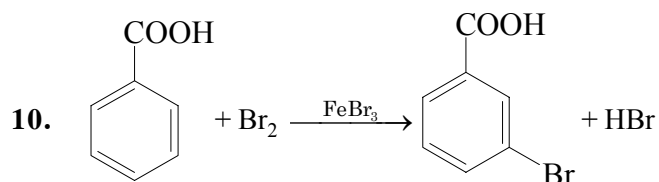
Based on these readings, and convention of titrimetric estimation of concentration of  $\text{Na}_2\text{CO}_3$  solution is \_\_\_\_\_ mM.

(Round off to the Nearest integer)

**Official Ans. by NTA (50)**

- Sol.** Most precise volume of HCl = 5 ml at equivalence point  
 Meq. of  $\text{Na}_2\text{CO}_3$  = meq. of HCl.  
 Let molarity of  $\text{Na}_2\text{CO}_3$  solution = M, then  
 $M \times 10 \times 2 = 0.2 \times 5 \times 1$   
 $M = 0.05 \text{ mol / L}$   
 $= 0.05 \times 1000$   
 $= 50 \text{ mM}$

**Sol.** Moles of Benzoic acid =  $\frac{6.1}{122}$   
 = moles of m-bromobenzoic acid  
 So, weight of m-bromobenzoic acid  
 $= \frac{6.1}{122} \times 201 \text{ gm}$   
 $= 10.05 \text{ gm}$   
 $\% \text{ yield} = \frac{\text{Actual weight}}{\text{Theoretical weight}} \times 100$   
 $= \frac{7.8}{10.05} \times 100$   
 $= 77.61\%$



Consider the above reaction where 6.1 g of benzoic acid is used to get 7.8 g of m-bromo benzoic acid. The percentage yield of the product is \_\_\_\_\_.

(Round off to the Nearest integer)

[Given : Atomic masses : C = 12.0u, H : 1.0u, O : 16.0u, Br = 80.0 u]

**Official Ans. by NTA (78)**

# VnK Academy Thane

## FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18<sup>th</sup> March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

### MATHEMATICS

### TEST PAPER WITH SOLUTION

#### SECTION-A

1. Let  $y = y(x)$  be the solution of the differential equation  $\frac{dy}{dx} = (y+1)((y+1)e^{x^2/2} - x)$ ,  $0 < x < 2$ , with  $y(2) = 0$ . Then the value of  $\frac{dy}{dx}$  at  $x = 1$  is equal to :

- (1)  $-\frac{e^{3/2}}{(e^2+1)^2}$       (2)  $-\frac{2e^2}{(1+e^2)^2}$   
 (3)  $\frac{e^{5/2}}{(1+e^2)^2}$       (4)  $\frac{5e^{1/2}}{(e^2+1)^2}$

**Official Ans. by NTA (1)**

**Sol.** Let  $y + 1 = Y$

$$\therefore \frac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$$

Put  $-\frac{1}{Y} = k$

$$\Rightarrow \frac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$$

I.F. =  $e^{-\frac{x^2}{2}}$

$$\therefore k = (x+c)e^{x^2/2}$$

Put  $k = -\frac{1}{y+1}$

$$\therefore y+1 = -\frac{1}{(x+c)e^{x^2/2}} \quad \dots(i)$$

when  $x = 2$ ,  $y = 0$ , then  $c = -2 - \frac{1}{e^2}$

differentiate equation (i) & put  $x = 1$

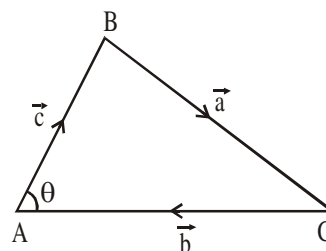
we get  $\left(\frac{dy}{dx}\right)_{x=1} = -\frac{e^{3/2}}{(1+e^2)^2}$

2. In a triangle ABC, if  $|\overrightarrow{BC}| = 8$ ,  $|\overrightarrow{CA}| = 7$ ,  $|\overrightarrow{AB}| = 10$ , then the projection of the vector  $\overrightarrow{AB}$  on  $\overrightarrow{AC}$  is equal to :

- (1)  $\frac{25}{4}$       (2)  $\frac{85}{14}$       (3)  $\frac{127}{20}$       (4)  $\frac{115}{16}$

**Official Ans. by NTA (2)**

**Sol.**



$$|\vec{a}| = 8, |\vec{b}| = 7, |\vec{c}| = 10$$

$$\cos \theta = \frac{|\vec{b}|^2 + |\vec{c}|^2 - |\vec{a}|^2}{2|\vec{b}||\vec{c}|} = \frac{17}{28}$$

Projection of  $\vec{c}$  on  $\vec{b}$

$$= |\vec{c}| \cos \theta$$

$$= 10 \times \frac{17}{28}$$

$$= \frac{85}{14}$$

3. Let the system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbb{R}.$$

has a non-trivial solution. Then which of the following is true ?

- (1)  $\mu = 6, \lambda \in \mathbb{R}$       (2)  $\lambda = 2, \mu \in \mathbb{R}$   
 (3)  $\lambda = 3, \mu \in \mathbb{R}$       (4)  $\mu = -6, \lambda \in \mathbb{R}$

**Official Ans. by NTA (1)**

**Sol.** For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$$

$$\text{when } \mu = 6, \quad 12 - 6\lambda + 6\lambda = 12$$

which is satisfied by all  $\lambda$

**4.** Let  $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{1\}$  be defined by

$$f(x) = \frac{x-2}{x-3}. \text{ Let } g : \mathbb{R} \rightarrow \mathbb{R} \text{ be given as}$$

$$g(x) = 2x - 3. \text{ Then, the sum of all the values}$$

$$\text{of } x \text{ for which } f^{-1}(x) + g^{-1}(x) = \frac{13}{2} \text{ is equal to}$$

- (1) 7      (2) 2      (3) 5      (4) 3

**Official Ans. by NTA (3)**

**Sol.**  $f(x) = y = \frac{x-2}{x-3}$

$$\therefore x = \frac{3y-2}{y-1}$$

$$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$$

$$\& g(x) = y = 2x - 3$$

$$\therefore x = \frac{y+3}{2}$$

$$\therefore g^{-1}(x) = \frac{x+3}{2}$$

$$\therefore f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$$

$$\therefore x^2 - 5x + 6 = 0 \begin{cases} x_1 \\ x_2 \end{cases}$$

$\therefore$  sum of roots

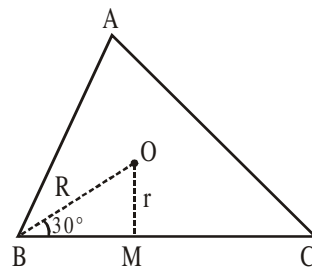
$$x_1 + x_2 = 5$$

**5.** Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line  $x + y = 3$ . If  $R$  and  $r$  be the radius of circumcircle and incircle respectively of  $\Delta ABC$ , then  $(R + r)$  is equal to :

- (1)  $\frac{9}{\sqrt{2}}$       (2)  $7\sqrt{2}$       (3)  $2\sqrt{2}$       (4)  $3\sqrt{2}$

**Official Ans. by NTA (1)**

**Sol.**



$$r = OM = \frac{3}{\sqrt{2}}$$

$$\& \sin 30^\circ = \frac{1}{2} = \frac{r}{R} \Rightarrow R = \frac{6}{\sqrt{2}}$$

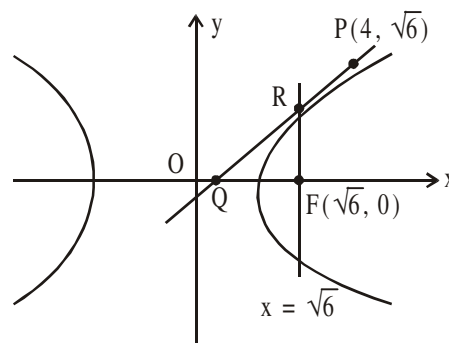
$$\therefore r + R = \frac{9}{\sqrt{2}}$$

**6.** Consider a hyperbola  $H : x^2 - 2y^2 = 4$ . Let the tangent at a point  $P(4, \sqrt{6})$  meet the x-axis at  $Q$  and latus rectum at  $R(x_1, y_1)$ ,  $x_1 > 0$ . If  $F$  is a focus of  $H$  which is nearer to the point  $P$ , then the area of  $\Delta QFR$  is equal to

- (1)  $4\sqrt{6}$       (2)  $\sqrt{6} - 1$   
(3)  $\frac{7}{\sqrt{6}} - 2$       (4)  $4\sqrt{6} - 1$

**Official Ans. by NTA (3)**

**Sol.**



$$\frac{x^2}{4} - \frac{y^2}{2} = 1$$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{3}{2}}$$

$$\therefore \text{Focus } F(ae, 0) \Rightarrow F(\sqrt{6}, 0)$$

equation of tangent at P to the hyperbola is

$$2x - y\sqrt{6} = 2$$

tangent meet x-axis at Q(1, 0)

$$\& \text{ latus rectum } x = \sqrt{6} \text{ at } R\left(\sqrt{6}, \frac{2}{\sqrt{6}}(\sqrt{6}-1)\right)$$

$$\therefore \text{Area of } \Delta_{QFR} = \frac{1}{2}(\sqrt{6}-1) \cdot \frac{2}{\sqrt{6}}(\sqrt{6}-1)$$

$$= \frac{7}{\sqrt{6}} - 2$$

7. If P and Q are two statements, then which of the following compound statement is a tautology ?

- (1)  $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$   
 (2)  $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$   
 (3)  $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$   
 (4)  $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

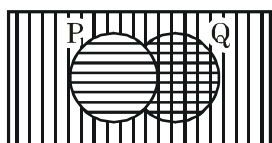
**Official Ans. by NTA (2)**

**Sol.** LHS of all the options are same i.e.

$$\begin{aligned} & ((P \Rightarrow Q) \wedge \sim Q) \\ & \equiv (\sim P \vee Q) \wedge \sim Q \\ & \equiv (\sim P \wedge \sim Q) \vee (Q \wedge \sim Q) \\ & \equiv \sim P \wedge \sim Q \end{aligned}$$

$$\begin{aligned} \text{(A)} \quad & (\sim P \wedge \sim Q) \rightarrow Q \\ & \equiv \sim(\sim P \wedge \sim Q) \vee Q \\ & \equiv (P \vee Q) \vee Q \neq \text{tautology} \end{aligned}$$

$$\begin{aligned} \text{(B)} \quad & (\sim P \wedge \sim Q) \rightarrow \sim P \\ & \equiv \sim(\sim P \wedge \sim Q) \vee \sim P \\ & \equiv (P \vee Q) \vee \sim P \end{aligned}$$



$\Rightarrow$  Tautology

$$\text{(C)} \quad (\sim P \wedge \sim Q) \rightarrow P$$

$$\equiv (P \vee Q) \vee P \neq \text{Tautology}$$

$$\text{(D)} \quad (\sim P \wedge \sim Q) \rightarrow (P \wedge Q)$$

$$\equiv (P \vee Q) \vee (P \wedge Q) \neq \text{Tautology}$$

**Aliter :**

P	Q	$P \vee Q$	$P \wedge Q$	$\sim P$	$(P \vee Q) \vee \sim P$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T

8. Let  $g(x) = \int_0^x f(t) dt$ , where  $f$  is continuous

function in  $[0, 3]$  such that  $\frac{1}{3} \leq f(t) \leq 1$  for all

$t \in [0, 1]$  and  $0 \leq f(t) \leq \frac{1}{2}$  for all  $t \in (1, 3]$ .

The largest possible interval in which  $g(3)$  lies is :

- (1)  $\left[-1, -\frac{1}{2}\right]$  (2)  $\left[-\frac{3}{2}, -1\right]$   
 (3)  $\left[\frac{1}{3}, 2\right]$  (4)  $[1, 3]$

**Official Ans. by NTA (3)**

$$\text{Sol.} \quad \frac{1}{3} \leq f(t) \leq 1 \quad \forall t \in [0, 1]$$

$$0 \leq f(t) \leq \frac{1}{2} \quad \forall t \in (1, 3]$$

$$\text{Now, } g(3) = \int_0^3 f(t) dt = \int_0^1 f(t) dt + \int_1^3 f(t) dt$$

$$\therefore \int_0^1 \frac{1}{3} dt \leq \int_0^1 f(t) dt \leq \int_0^1 1 dt \quad \dots (1)$$

$$\text{and } \int_1^3 0 dt \leq \int_1^3 f(t) dt \leq \int_1^3 \frac{1}{2} dt \quad \dots (2)$$

Adding, we get

$$\frac{1}{3} + 0 \leq g(3) \leq 1 + \frac{1}{2}(3-1)$$

$$\frac{1}{3} \leq g(3) \leq 2$$

9. Let  $S_1$  be the sum of first  $2n$  terms of an arithmetic progression. Let  $S_2$  be the sum of first  $4n$  terms of the same arithmetic progression. If  $(S_2 - S_1)$  is 1000, then the sum of the first  $6n$  terms of the arithmetic progression is equal to:  
(1) 1000 (2) 7000 (3) 5000 (4) 3000

**Official Ans. by NTA (4)**

**Sol.**  $S_{2n} = \frac{2n}{2} [2a + (2n-1)d]$ ,  $S_{4n} = \frac{4n}{2} [2a + (4n-1)d]$

$$\Rightarrow S_2 - S_1 = \frac{4n}{2} [2a + (4n-1)d] - \frac{2n}{2} [2a + (2n-1)d]$$

$$1)d]$$

$$= 4an + (4n-1)2nd - 2na - (2n-1)dn$$

$$= 2na + nd[8n-2-2n+1]$$

$$\Rightarrow 2na + 2n[6n-1] = 1000$$

$$2a + (6n-1)d = \frac{1000}{n}$$

Now,  $S_{6n} = \frac{6n}{2} [2a + (6n-1)d]$

$$= 3n \cdot \frac{1000}{n} = 3000$$

10. Let a complex number be  $w = 1 - \sqrt{3}i$ . Let another complex number  $z$  be such that  $|zw| = 1$

and  $\arg(z) - \arg(w) = \frac{\pi}{2}$ . Then the area of the

triangle with vertices origin,  $z$  and  $w$  is equal to :

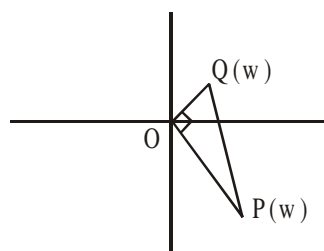
(1) 4 (2)  $\frac{1}{2}$  (3)  $\frac{1}{4}$  (4) 2

**Official Ans. by NTA (2)**

**Sol.**  $w = 1 - \sqrt{3}i \Rightarrow |w| = 2$

Now,  $|z| = \frac{1}{|w|} \Rightarrow |z| = \frac{1}{2}$

and  $\arg(z) = \frac{\pi}{2} + \arg(w)$



$$\Rightarrow \text{Area of triangle} = \frac{1}{2} \cdot OP \cdot OQ$$

$$= \frac{1}{2} \cdot 2 \cdot \frac{1}{2} = \frac{1}{2}$$

11. Let in a series of  $2n$  observations, half of them are equal to  $a$  and remaining half are equal to  $-a$ . Also by adding a constant  $b$  in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of  $a^2 + b^2$  is equal to :

(1) 425 (2) 650 (3) 250 (4) 925

**Official Ans. by NTA (1)**

- Sol.** Let observations are denoted by  $x_i$  for  $1 \leq i < 2n$

$$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a + a + \dots + a) - (a + a + \dots + a)}{2n}$$

$$\Rightarrow \bar{x} = 0$$

$$\text{and } \sigma_x^2 = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \frac{a^2 + a^2 + \dots + a^2}{2n} - 0 = a^2$$

$$\Rightarrow \sigma_x = a$$

Now, adding a constant  $b$  then  $\bar{y} = \bar{x} + b = 5$

$$\Rightarrow b = 5$$

and  $\sigma_y = \sigma_x$  (No change in S.D.)  $\Rightarrow a = 20$

$$\Rightarrow a^2 + b^2 = 425$$

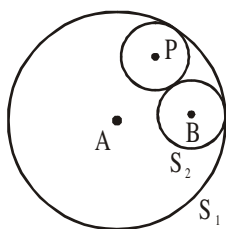
12. Let  $S_1 : x^2 + y^2 = 9$  and  $S_2 : (x - 2)^2 + y^2 = 1$ . Then the locus of center of a variable circle  $S$  which touches  $S_1$  internally and  $S_2$  externally always passes through the points :

- (1)  $(0, \pm\sqrt{3})$  (2)  $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$   
 (3)  $\left(2, \pm\frac{3}{2}\right)$  (4)  $(1, \pm 2)$

**Official Ans. by NTA (3)**

**Sol.**  $S_1 : x^2 + y^2 = 9 \begin{cases} r_1 = 3 \\ A(0, 0) \end{cases}$   
 $S_2 : (x - 2)^2 + y^2 = 1 \begin{cases} r_2 = 1 \\ B(2, 0) \end{cases}$

$\therefore c_1 c_2 = r_1 - r_2$



$\therefore$  given circle are touching internally

Let a variable circle with centre  $P$  and radius  $r$

$\Rightarrow PA = r_1 - r$  and  $PB = r_2 + r$

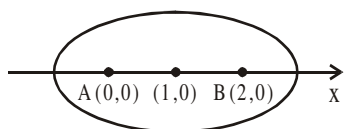
$\Rightarrow PA + PB = r_1 + r_2$

$\Rightarrow PA + PB = 4$  ( $> AB$ )

$\Rightarrow$  Locus of  $P$  is an ellipse with foci at  $A(0, 0)$  and  $B(2, 0)$  and length of major axis is  $2a = 4$ ,

$e = \frac{1}{2}$

$\Rightarrow$  centre is at  $(1, 0)$  and  $b^2 = a^2(1 - e^2) = 3$  if  $x$ -ellipse



$\Rightarrow E: \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$

which is satisfied by  $\left(2, \pm\frac{3}{2}\right)$

13. Let  $\vec{a}$  and  $\vec{b}$  be two non-zero vectors perpendicular to each other and  $|\vec{a}| = |\vec{b}|$ . If

$|\vec{a} \times \vec{b}| = |\vec{a}|$ , then the angle between the vectors

$(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$  and  $\vec{a}$  is equal to :

(1)  $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$  (2)  $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3)  $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$  (4)  $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

**Official Ans. by NTA (2)**

**Sol.**  $|\vec{a}| = |\vec{b}|$ ,  $|\vec{a} \times \vec{b}| = |\vec{a}|$ ,  $\vec{a} \perp \vec{b}$

$|\vec{a} \times \vec{b}| = |\vec{a}| \Rightarrow |\vec{a}||\vec{b}|\sin 90^\circ = |\vec{a}| \Rightarrow |\vec{b}| = 1 = |\vec{a}|$

$\vec{a}$  and  $\vec{b}$  are mutually perpendicular unit vectors.

Let  $\vec{a} = \hat{i}$ ,  $\vec{b} = \hat{j} \Rightarrow \vec{a} \times \vec{b} = \hat{k}$

$\cos \theta = \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot \hat{i}}{\sqrt{3} \sqrt{1}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

14. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1)  $\frac{32}{625}$  (2)  $\frac{80}{243}$  (3)  $\frac{40}{243}$  (4)  $\frac{128}{625}$

**Official Ans. by NTA (1)**

**Sol.**  $P(X = 1) = {}^5C_1 \cdot p \cdot q^4 = 0.4096$

$P(X = 2) = {}^5C_2 \cdot p^2 \cdot q^3 = 0.2048$

$\Rightarrow \frac{q}{2p} = 2$

$\Rightarrow q = 4p$  and  $p + q = 1$

$\Rightarrow p = \frac{1}{5}$  and  $q = \frac{4}{5}$

Now

$P(X = 3) = {}^5C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^2 = \frac{10 \times 16}{125 \times 25} = \frac{32}{625}$

15. Let a tangent be drawn to the ellipse  $\frac{x^2}{27} + y^2 = 1$

at  $(3\sqrt{3}\cos\theta, \sin\theta)$  where  $\theta \in \left(0, \frac{\pi}{2}\right)$ . Then the

value of  $\theta$  such that the sum of intercepts on axes made by this tangent is minimum is equal to :

- (1)  $\frac{\pi}{8}$       (2)  $\frac{\pi}{4}$       (3)  $\frac{\pi}{6}$       (4)  $\frac{\pi}{3}$

**Official Ans. by NTA (3)**

**Sol.** Equation of tangent be

$$\frac{x \cos \theta}{3\sqrt{3}} + \frac{y \sin \theta}{1} = 1, \quad \theta \in \left(0, \frac{\pi}{2}\right)$$

intercept on x-axis

$$OA = 3\sqrt{3} \sec \theta$$

intercept on y-axis

$$OB = \operatorname{cosec} \theta$$

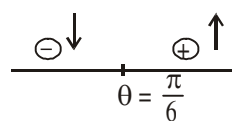
Now, sum of intercept

$$= 3\sqrt{3} \sec \theta + \operatorname{cosec} \theta = f(\theta) \text{ let}$$

$$f'(\theta) = 3\sqrt{3} \sec \theta \tan \theta - \operatorname{cosec} \theta \cot \theta$$

$$= 3\sqrt{3} \frac{\sin \theta}{\cos^2 \theta} - \frac{\cos \theta}{\sin^2 \theta}$$

$$= \underbrace{\frac{\cos \theta}{\sin^2 \theta}}_{\oplus} \cdot 3\sqrt{3} \left[ \tan^2 \theta - \frac{1}{3\sqrt{3}} \right] = 0 \Rightarrow \theta = \frac{\pi}{6}$$



$\Rightarrow$  at  $\theta = \frac{\pi}{6}$ ,  $f(\theta)$  is minimum

16. Define a relation  $R$  over a class of  $n \times n$  real matrices  $A$  and  $B$  as " $ARB$  iff there exists a non-singular matrix  $P$  such that  $PAP^{-1} = B$ ". Then which of the following is true ?
- (1)  $R$  is symmetric, transitive but not reflexive,
  - (2)  $R$  is reflexive, symmetric but not transitive
  - (3)  $R$  is an equivalence relation
  - (4)  $R$  is reflexive, transitive but not symmetric

**Official Ans. by NTA (3)**

**Sol.**  $A$  and  $B$  are matrices of  $n \times n$  order &  $ARB$  iff there exists a non singular matrix  $P$  ( $\det(P) \neq 0$ ) such that  $PAP^{-1} = B$

**For reflexive**

$$ARA \Rightarrow PAP^{-1} = A \quad \dots(1) \text{ must be true}$$

for  $P = I$ , Eq.(1) is true so ' $R$ ' is reflexive

**For symmetric**

$$ARB \Leftrightarrow PAP^{-1} = B \quad \dots(1) \text{ is true}$$

for  $BRA$  iff  $PBP^{-1} = A \quad \dots(2) \text{ must be true}$

$$\therefore PAP^{-1} = B$$

$$P^{-1}PAP^{-1} = P^{-1}B$$

$$IAP^{-1}P = P^{-1}BP$$

$$A = P^{-1}BP \quad \dots(3)$$

$$\text{from (2) \& (3) } PBP^{-1} = P^{-1}BP$$

can be true some  $P = P^{-1} \Rightarrow P^2 = I$  ( $\det(P) \neq 0$ )

So ' $R$ ' is symmetric

**For transitive**

$$ARB \Leftrightarrow PAP^{-1} = B \dots \text{ is true}$$

$$BRC \Leftrightarrow PBP^{-1} = C \dots \text{ is true}$$

$$\text{now } PPAP^{-1}P^{-1} = C$$

$$P^2A(P^2)^{-1} = C \Rightarrow ARC$$

So ' $R$ ' is transitive relation

$\Rightarrow$  Hence  $R$  is equivalence

17. A pole stands vertically inside a triangular park  $ABC$ . Let the angle of elevation of the top of

the pole from each corner of the park be  $\frac{\pi}{3}$ .

If the radius of the circumcircle of  $\Delta ABC$  is 2, then the height of the pole is equal to :

- (1)  $\frac{2\sqrt{3}}{3}$       (2)  $2\sqrt{3}$       (3)  $\sqrt{3}$       (4)  $\frac{1}{\sqrt{3}}$

**Official Ans. by NTA (2)**

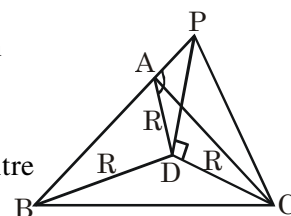
**Sol.** Let  $PD = h$ ,  $R = 2$

As angle of elevation

of top of pole from

$A$ ,  $B$ ,  $C$  are equal So

$D$  must be circumcentre of  $\Delta ABC$



$$\tan\left(\frac{\pi}{3}\right) = \frac{PD}{R} = \frac{h}{R}$$

$$h = R \tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$

18. If  $15\sin^4\alpha + 10\cos^4\alpha = 6$ , for some  $\alpha \in \mathbb{R}$ , then the value of  $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$  is equal to :  
 (1) 350 (2) 500 (3) 400 (4) 250

**Official Ans. by NTA (4)**

**Sol.**  $15\sin^4\alpha + 10\cos^4\alpha = 6$   
 $15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$   
 $(3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$

$$\tan^2\alpha = \frac{2}{3} \cdot \cot^2\alpha = \frac{3}{2}$$

$$\Rightarrow 27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$$

$$= 27(\sec^6\alpha)^3 + 8(\operatorname{cosec}^6\alpha)^3$$

$$= 27(1 + \tan^2\alpha)^3 + 8(1 + \cot^2\alpha)^3$$

$$= 250$$

19. The area bounded by the curve  $4y^2 = x^2(4-x)(x-2)$  is equal to :

(1)  $\frac{\pi}{8}$  (2)  $\frac{3\pi}{8}$  (3)  $\frac{3\pi}{2}$  (4)  $\frac{\pi}{16}$

**Official Ans. by NTA (3)**

**Sol.**  $4y^2 = x^2(4-x)(x-2)$

$$|y| = \frac{|x|}{2} \sqrt{(4-x)(x-2)}$$

$$\Rightarrow y_1 = \frac{x}{2} \sqrt{(4-x)(x-2)}$$

$$\text{and } y_2 = \frac{-x}{2} \sqrt{(4-x)(x-2)}$$

$$D : x \in [2, 4]$$

Required Area

$$= \int_2^4 (y_1 - y_2) dx = \int_2^4 x \sqrt{(4-x)(x-2)} dx \quad \dots(1)$$

$$\text{Applying } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

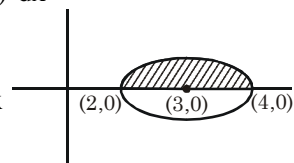
$$\text{Area} = \int_2^4 (6-x) \sqrt{(4-x)(x-2)} dx \quad \dots(2)$$

$$(1) + (2)$$

$$2A = 6 \int_2^4 \sqrt{(4-x)(x-2)} dx$$

$$A = 3 \int_2^4 \sqrt{1-(x-3)^2} dx$$

$$A = 3 \cdot \frac{\pi}{2} \cdot 1^2 = \frac{3\pi}{2}$$



20. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x} & , \text{ if } x < 0 \\ b & , \text{ if } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{ if } x > 0 \end{cases}$$

If  $f$  is continuous at  $x = 0$ , then the value of  $a + b$  is equal to :

(1)  $-\frac{5}{2}$  (2)  $-2$  (3)  $-3$  (4)  $-\frac{3}{2}$

**Official Ans. by NTA (4)**

**Sol.**  $f(x)$  is continuous at  $x = 0$

$$\lim_{x \rightarrow 0^+} f(x) = f(0) = \lim_{x \rightarrow 0^-} f(x) \quad \dots(1)$$

$$f(0) = b \quad \dots(2)$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left( \frac{\sin(a+1)x}{2x} + \frac{\sin 2x}{2x} \right)$$

$$= \frac{a+1}{2} + 1 \quad \dots(3)$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x \rightarrow 0^+} \frac{(x+bx^3-x)}{bx^{5/2}(\sqrt{x+bx^3} + \sqrt{x})}$$

$$= \lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sqrt{x}(\sqrt{1+bx^2} + 1)} = \frac{1}{2} \quad \dots(4)$$

Use (2), (3) & (4) in (1)

$$\frac{1}{2} = b = \frac{a+1}{2} + 1$$

$$\Rightarrow b = \frac{1}{2}, a = -2$$

$$a + b = \frac{-3}{2}$$



1. If  $f(x)$  and  $g(x)$  are two polynomials such that the polynomial  $P(x) = f(x^3) + xg(x^3)$  is divisible by  $x^2 + x + 1$ , then  $P(1)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (0)**

**Sol.**  $P(x) = f(x^3) + xg(x^3)$   
 $P(1) = f(1) + g(1) \quad \dots(1)$   
 Now  $P(x)$  is divisible by  $x^2 + x + 1$   
 $\Rightarrow P(x) = Q(x)(x^2 + x + 1)$   
 $P(w) = 0 = P(w^2)$  where  $w, w^2$  are non-real cube roots of units  
 $P(x) = f(x^3) + xg(x^3)$   
 $P(w) = f(w^3) + wg(w^3) = 0$   
 $f(1) + wg(1) = 2 \quad \dots(2)$   
 $P(w^2) = f(w^6) + w^2g(w^6) = 0$   
 $f(1) + w^2g(1) = 0 \quad \dots(3)$   
 $(2) + (3)$   
 $\Rightarrow 2f(1) + (w + w^2)g(1) = 0$   
 $2f(1) = g(1) \quad \dots(4)$   
 $(2) - (3)$   
 $\Rightarrow (w - w^2)g(1) = 0$   
 $g(1) = 0 = f(1)$  from (4)  
 from (1)  $P(1) = f(1) + g(1) = 0$

2. Let  $I$  be an identity matrix of order  $2 \times 2$  and

$$P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}. \text{ Then the value of } n \in \mathbb{N} \text{ for}$$

which  $P^n = 5I - 8P$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (6)**

**Sol.**  $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n$$

$$\Rightarrow n = 6$$

3. If  $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$ , then the value of  $\alpha$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (160)**

**Sol.**  $\sum_{r=1}^{10} r!\{(r+1)(r+2)(r+3) - 9(r+1) + 8\}$   
 $= \sum_{r=1}^{10} [ \{(r+3)! - (r+1)!\} - 8\{(r+1)! - r!\} ]$   
 $= (13! + 12! - 2! - 3!) - 8(11! - 1)$   
 $= (12 \cdot 13 + 12 - 8) \cdot 11! - 8 + 8$   
 $= (160)(11!)$   
 Hence  $\alpha = 160$

4. The term independent of  $x$  in the expansion of

$$\left[ \frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right]^{10}, x \neq 1, \text{ is equal to}$$

\_\_\_\_\_.

**Official Ans. by NTA (210)**

**Sol.**  $\left( (x^{1/3} + 1) - \left( \frac{\sqrt{x} + 1}{\sqrt{x}} \right) \right)^{10}$

$$(x^{1/3} - x^{-1/2})^{10}$$

$$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r$$

$$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$$

$$\Rightarrow r = 4$$

$$T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

5. Let  $P(x)$  be a real polynomial of degree 3 which vanishes at  $x = -3$ . Let  $P(x)$  have local minima at  $x = 1$ , local maxima at  $x = -1$  and

$$\int_{-1}^1 P(x) dx = 18, \text{ then the sum of all the}$$

coefficients of the polynomial  $P(x)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (8)**

**Sol.** Let  $p'(x) = a(x-1)(x+1) = a(x^2-1)$

$$p(x) = a \int (x^2-1) dx + c$$

$$= a \left( \frac{x^3}{3} - x \right) + c$$

$$\text{Now } p(-3) = 0$$

$$\Rightarrow a \left( -\frac{27}{3} + 3 \right) + c = 0$$

$$\Rightarrow -6a + c = 0 \quad \dots(1)$$

$$\text{Now } \int_{-1}^1 \left( a \left( \frac{x^3}{3} - x \right) + c \right) dx = 18$$

$$= 2c = 18 \Rightarrow c = 9 \quad \dots(2)$$

$$\Rightarrow \text{from (1) \& (2)} \Rightarrow -6a + 9 = 0 \Rightarrow a = \frac{3}{2}$$

$$\Rightarrow p(x) = \frac{3}{2} \left( \frac{x^3}{3} - x \right) + 9$$

sum of coefficient

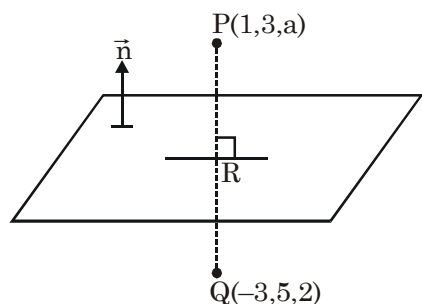
$$= \frac{1}{2} - \frac{3}{2} + 9$$

$$= 8$$

6. Let the mirror image of the point  $(1, 3, a)$  with respect to the plane  $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$  be  $(-3, 5, 2)$ . Then the value of  $|a + b|$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (1)**

**Sol.**



$$\text{plane} = 2x - y + z = b$$

$$R \equiv \left( -1, 4, \frac{a+2}{2} \right) \rightarrow \text{on plane}$$

$$\therefore -2 - 4 + \frac{a+2}{2} = b$$

$$\Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10 \quad \dots(i)$$

$$\langle PQ \rangle = \langle 4, -2, a-2 \rangle$$

$$\therefore \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2}$$

$$\Rightarrow a - 2 = 2 \Rightarrow a = 4, b = -3$$

$$\therefore |a + b| = 1$$

7. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfy the equation  $f(x+y) = f(x) \cdot f(y)$  for all  $x, y \in \mathbb{R}$  and  $f(x) \neq 0$  for any  $x \in \mathbb{R}$ . If the function  $f$  is differentiable at  $x = 0$  and  $f'(0) = 3$ , then

$$\lim_{h \rightarrow 0} \frac{1}{h} (f(h) - 1) \text{ is equal to } \underline{\hspace{2cm}}.$$

**Official Ans. by NTA (3)**

**Sol.** If  $f(x+y) = f(x) \cdot f(y)$  &  $f'(0) = 3$  then

$$f(x) = a^x \Rightarrow f'(x) = a^x \cdot \ln a$$

$$\Rightarrow f'(0) = \ln a = 3 \Rightarrow a = e^3$$

$$\Rightarrow f(x) = (e^3)^x = e^{3x}$$

$$\lim_{x \rightarrow 0} \frac{f(x) - 1}{x} = \lim_{x \rightarrow 0} \left( \frac{e^{3x} - 1}{3x} \times 3 \right) = 1 \times 3 = 3$$

8. Let  ${}^nC_r$  denote the binomial coefficient of  $x^r$  in the expansion of  $(1+x)^n$ .

$$\text{If } \sum_{k=0}^{10} (2^2 + 3k) {}^nC_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}, \alpha, \beta \in \mathbb{R},$$

then  $\alpha + \beta$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (19)**

**Allen Answer (Bonus)**

**Sol. BONUS**

Instead of  ${}^nC_k$  it must be  ${}^{10}C_k$  i.e.

$$\sum_{k=0}^{10} (2^2 + 3k) {}^{10}C_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$$

$$\text{LHS} = 4 \sum_{k=0}^{10} {}^{10}C_k + 3 \sum_{k=0}^{10} k \cdot \frac{10}{k} {}^9C_{k-1}$$

$$= 4 \cdot 2^{10} + 3 \cdot 10 \cdot 2^9$$

$$= 19 \cdot 2^{10} = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$$

$$\Rightarrow \alpha = 0, \beta = 19 \Rightarrow \alpha + \beta = 19$$

9. Let P be a plane containing the line

$$\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2} \text{ and parallel to the line}$$

$$\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}. \text{ If the point } (1, -1, \alpha) \text{ lies}$$

on the plane P, then the value of  $5\alpha$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (38)**

**Sol.** Equation of plane is  $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

Now  $(1, -1, \alpha)$  lies on it so

$$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0 \Rightarrow 5\alpha + 38 = 0 \Rightarrow 5\alpha = -38$$

10. Let  $y = y(x)$  be the solution of the differential equation  $xdy - ydx = \sqrt{x^2 - y^2} dx$ ,  $x \geq 1$ , with  $y(1) = 0$ . If the area bounded by the line  $x = 1$ ,  $x = e^\pi$ ,  $y = 0$  and  $y = y(x)$  is  $\alpha e^{2\pi} + \beta$ , then the value of  $10(\alpha + \beta)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (4)**

**Sol.**  $xdy - ydx = \sqrt{x^2 - y^2} dx$

$$\Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x} \sqrt{1 - \frac{y^2}{x^2}} dx$$

$$\Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ln|x| + c$$

$$\text{at } x = 1, y = 0 \Rightarrow c = 0$$

$$y = x \sin(\ln x)$$

$$A = \int_1^{e^\pi} x \sin(\ln x) dx$$

$$x = e^t, dx = e^t dt \Rightarrow \int_0^\pi e^{2t} \sin(t) dt = A$$

$$\alpha e^{2\pi} + \beta = \left( \frac{e^{2t}}{5} (2 \sin t - \cos t) \right)_0^\pi = \frac{1 + e^{2\pi}}{5}$$

$$\alpha = \frac{1}{5}, \beta = \frac{1}{5} \text{ so } 10(\alpha + \beta) = 4$$